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Designating the right member of this inequality by u_{r+1} , we see that Σu_r is convergent, since

$$\lim_{r=\infty} \frac{u_{r+1}}{u_r} = \lim_{r=\infty} \frac{9}{10} \left(\frac{r+1}{r} \right)^{p-1} = \frac{9}{10} < 1.$$

Hence Σg_r is convergent, and therefore also S is convergent.

NOTE.—One may generalize the proposition thus:

The series S' whose terms are the reciprocals of all positive integers not containing any combination C whatever of the digits 0, 1, 2, $\dots$, 9 (which contains at least one digit) is convergent.

For, suppose C contains p digits. It may be shown that there are not more terms in a group g'_{r+1} in this series than in the corresponding group g_{r+1} previously discussed. When this is established the preceding argument proves the proposition.

When the terms of the series are the reciprocals of all the positive integers the series diverges. The proposition states that if from this divergent series certain terms are stricken out a convergent series is obtained. This is not particularly surprising when one observes that obviously a very large percentage of the integers of a thousand places, for example, contain a zero, and hence most of the terms far out in the series are stricken out.

NOTE.—This solution was received before the appearance of the article by Dr. Irwin on "A curious convergent series" in the May MONTHLY. Dr. Irwin there proves this generalized theorem by a somewhat different style of argument. He gives on page 50 some details of the reasoning by which equation (1) above may be established.—EDITORS.

Also solved by A. H. HOLMES and G. W. HARTWELL.

454. Proposed by C. N. SCHMALL, New York City.

Prove that a number is divisible by nine if, and only if, the sum of its digits is divisible by nine.

SOLUTION BY FRANK R. MORRIS, Glendale, California.

Let $a, b, c, d, \dots$ be the digits of any whole number, in order from right to left. Then this number may be written in the form

$$a + 10b + 100c + 1,000d + \dots$$

Dividing by 9 the quotient is

$$a/9 + (1 + 1/9)b + (11 + 1/9)c + (111 + 1/9)d + \dots,$$

which equals

$$(b + 11c + 111d \dots) + \frac{a + b + c + d + \dots}{9}.$$

The first term of this quotient is integral and the second term is integral only if $a + b + c + d + \dots$ is divisible by 9. Therefore, the number is divisible by nine if, and only if, the sum of its digits is divisible by nine.

This proof holds for a number containing a decimal fraction, as may be seen by converting the number into a whole number divided by some power of ten.

The above number may also be written

$$a + (9 + 1)b + (99 + 1)c + (999 + 1)d + \dots$$

or

$$9b + 99c + 999d + \dots + (a + b + c + d + \dots),$$

which is a multiple of 9 when $(a + b + c + d + \dots)$ is a multiple of 9.

Also solved by PAUL CAPRON, ELIJAH SWIFT, J. W. CLAWSON, G. L. WAGAR, H. C. FEEMSTER, W. F. RIGGS, NATHAN ALTSHILLER, E. F. CANADAY, C. A. BARNHART, O. S. ADAMS, G. W. HARTWELL, A. W. SMITH, E. E. WHITFORD, HORACE OLSON, A. H. HOLMES, W. J. THOME, H. S. UHLER, and H. N. CARLETON.

GEOMETRY.

475. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Given two circles and a straight line, to draw a circle tangent to the line and coaxial with the two given circles.